

## RESEARCH ARTICLE

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## Parallel Machine Scheduling with Controllable Processing Time Considering Energy Cost and Machine Failure Prediction

Youssef Rabbani <sup>1\*</sup>, Ali Qorbani <sup>2</sup>, Reza Kamran Rad <sup>3</sup>**Abstract**

Predicting unexpected incidents and energy consumption decline is one of the current problems in the industry. The extant study addressed parallel machine scheduling by consideration of failures and energy consumption decline. Moreover, the present paper aimed at minimizing early and late delivery penalties, and enhancing tasks. This research designed a mathematical model for this problem that considered processing times, delivery time, rotation speed and torque, failure time, and machine availability after repair and maintenance. Failure times have been predicated on using machine learning algorithms. The results indicated that the proposed model can be suitably solved for the size of 10 jobs or tasks and five machines. This research addresses the problem in two parts: the first part predicts failures, and the second part includes the sequence of parallel machine scheduling operations. After the previous data were received in the first step, machine failure was predicted by using machine learning algorithms, and a set of rules were obtained to correct the process. The obtained rules were used in the model to improve the machining process. In the second step, scheduling mode was used to determine operations sequence by consideration of these failures and machinery unavailability to achieve the optimal sequence. Moreover, it is supposed to reduce energy consumption and failures. This study used the Light GBM algorithm and achieved 85% precision in failure prediction. The rules obtained from this algorithm contributed to cost reduction.

**Keywords:** *Machine Failure, Parallel Machines Scheduling, Data Mining, Just-in-Time Delivery, Energy*

**Introduction**

Global warming and climate change threaten life on the earth. The mentioned changes have occurred because of human activities, which lead to the emission of greenhouse gases. Intensive industrial activities and fuel energy overuse have led to this issue (Hidri et al., 2021). Now, the manufacturing industry almost consumes half of the energy produced on the planet earth. Optimal planning is the core case because of the energy products that can achieve such planning (Nicolo et al., 2019, Rabbani, 2021).

On the other hand, various mathematical models and optimization methods have been proposed to solve the problem of scheduling parallel machines by consideration of minimizing energy consumption. (Salimifard et al., 2019) studied parallel machine scheduling by considering job processing and running times, which are not constant times. An important task in production planning is manufacturing products regularly based on precise planning for just-in-time delivery of products to customers. The early or late

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delivery process may cause some problems, so this is a critical issue. Moreover, prediction of unexpected incidents is one of the common problems in industry, which harms production, and machine failure that changes the scheduling process may change the production structure, and increase cost and energy consumption. In the data mining process, we consider some factors, such as air temperature, process temperature, tool cycle speed, torque power, tool wear, accidental breakdowns, and power outage to overcome machine failure. We use data mining algorithms to predict these disorders in the future. Furthermore, it is possible to consider the time and date of repair and maintenance intervention by using these algorithms. Therefore, the extant study aims to predict these failures and disorders to save energy and deliver the product on time. We apply a specific mathematical model to solve smaller dimensions of this timing problem. Just-in-time delivery and lower energy consumption are objectives of this model. It is assumed that a set of  $n$ -member independent jobs exists  $N = \{j_1, j_2, \dots, j_n\}$  that is processed in parallel machines. All machines can process all jobs, and each machine can do a maximum of one job simultaneously. All jobs are available at time zero and are processed only

with a specific machine. The processing time can be compressed or expanded as much as it requires compression or expansion cost. The extra processing cost is needed to process a job in normal time. Because all machines are identical, the processing time of each job on every machine is identical. Moreover, it is not allowed to prefer the priority of a piece for selection, and the number of jobs and machines is fixed. Machine running time and transfer time between machines are very low. The mathematical model has been proposed to decrease total costs and failures. In this way, data mining helps to find some rules about the torque speed in which, the machine fails, and then we can apply relevant constraints to prevent the machine go beyond its allowed extent for compressing and enhancing jobs. Moreover, the availability or unavailability times of the machine are inserted into the mathematical model using prediction results to schedule the problem based on the parameters, including rotational speed, torque, processing time, etc. This approach tends to achieve a just-in-time and inexpensive sequence of operations to optimize both energy consumption and the machining process. Also, Figure 1 is an overview of the overall structure of the model.

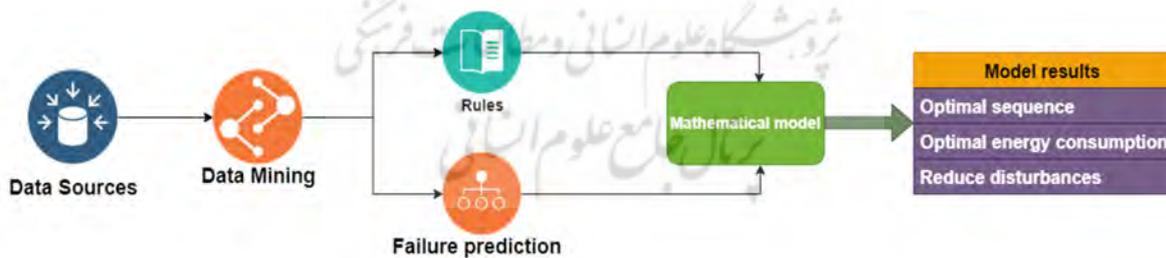


Figure 1. Structure of the model

## 1. Literature Review

Because the present study aims at optimizing parallel machine scheduling by reducing energy consumption, which was expressed by (Hidri et al., 2021). Hence, this study tends to find how we can predict the failure of the machine in the real world by

using available tools. This study also addresses how to achieve the optimal sequence of a set of processable jobs on the machines in the existing conditions. Some studies conducted in this context have been reviewed in three sections herein.

### 1.1. Parallel machine Scheduling based on a just-in-time delivery approach

Some papers have simultaneously studied the on-time delivery and delay criteria in parallel machines to find the optimal sequences in scheduling problems with just-in-time (JIT) delivery. (Nowicki & Zdrzałka, 1990) studied scheduling machines with controllable processing time and linear cost functions. (Kayvanfar et al., 2017) addressed the problem of scheduling identical parallel machines with controllable processing times by using JIT delivery problem-solving. They proposed a mixed integer linear programming model, used Meta-heuristic Intelligent Water Drop Algorithm for small, medium, and large dimensions, and found acceptable results. (Zarandi & Kayvanfar, 2015) studied bi-objective scheduling problems with controllable processing times in identical parallel machines and pursued two objectives:

1) minimizing total cost of delay, early delivery, compression simultaneously, and expanding job processing times;

2) maximizing completion time. In addition to two meta-heuristic multi-objective algorithms of non-dominated ranking and non-dominated genetic sorting, they used a heuristic algorithm to measure optimal compression rate and processing time expansion.

(Su, 2009) studied the problem of identical parallel machine scheduling to minimize the

early and late delivery of products on a certain date regarding the total floating time. (Polyakovskiy & M'Hallah, 2014) examined product delivery delay by identical parallel devices, in which, jobs have various processing times and certain delivery deadlines. Theoretically, there is a significant association between scheduling problems considering early and late product delivery criteria and the concept of controllable processing time. The reason is that early or late delivery can be reduced by controlling jobs' process time approaching the system to the immediate delivery of the product to the customer. (Yazdani & Jolai, 2015) proposed A Genetic Algorithm with Modified Crossover Operator for a Two-Agent Scheduling Problem. Table 1 reports other relevant studies. (Dang et al., 2021) assessed the probability of rework, completion times, and other objectives in their problems. They proposed an integer small-sized linear programming model for this problem, while they used a hybrid genetic algorithm for larger dimensions. (Kramer et al., 2021) studied job scheduling problems on identical parallel machines with running time to minimize the job completion time in the weighted form. They proposed five new models to solve this problem. Table 1 reports other relevant studies.

Table 1

*Research background in terms of examined concepts and provided solution methods*

References	Explanations	Problem
(Arik & Toksari, 2018)	Considering deterioration effect, learning effect, and processing times in fuzzy mode and using the mathematical fuzzy model and local search algorithm	Scheduling multi-objective parallel machines
(Cheng & Huang, 2017)	Using integer linear programming model and modified genetic algorithm	parallel machine Scheduling by minimizing early and late delivery
(Goli & Keshavarz, 2021)	Using a mathematical model for small dimensions and using Biogeography-Based Optimization (BBO), variable neighborhood search (VNS), and tree search	parallel machine Scheduling by minimizing the weight of early and late delivery

References	Explanations	Problem
(Mohr et al., 2021)	Using a genetic algorithm and tree search	parallel machine Scheduling by minimizing total weighted flow time
(Kubiak, 1993)	Using a mathematical model with a JIT delivery approach	parallel machine Scheduling by minimizing variations in manufacturing systems
(Exposito-Izquierdo et al., 2019)	Using metaheuristic algorithm and multi-agent simulation	parallel machine Scheduling considering the learning effect

## 2.2. Parallel machine scheduling considering energy consumption constraints

(Wang et al., 2018) studied identical parallel machine scheduling to minimize total energy consumption and job completion time. They used Augmented Epsilon Constraint Method, heuristic method, and multi-objective Non-dominated Sorting Genetic Algorithm (NSGA-II) to solve problems with small, medium, and large dimensions, respectively. (Antoniadis et al., 2020) minimized energy consumption in scheduling parallel machines by consideration of release time, delivery time, and processing time parameters. They also considered machine sleep mode in which, the machine does not consume energy. (Nanthapodej et al., 2021) parallel machine scheduled to minimize energy consumption. They also aimed at improving efficiency by balancing the input load of the machine. They used a mathematical model to solve this problem, and employed a hybrid differential evolution algorithm to solve large and medium dimensions of this problem.

## 2.3. Parallel machine scheduling considering failures, maintenance, and repair

Studies on the effects of maintenance and repair types in machine scheduling have received great attention. In terms of predictor maintenance and repair, machine learning has become Common machine learning methods have been proposed for preventive maintenance and repair problems (Bilski, 2014; Calabrese et al., 2020; Schmidt & Wang, 2018). Machine learning methods consist of using some classifiers, such as support vector machines, decision trees, random forests, and Naive Bayes. Moreover, (Chen, 2009) widely

used the periodic repair and maintenance approach. (Wang & Liu, 2015) studied the problem of integrated production planning and preventive maintenance for parallel machine scheduling to minimize machine unavailability and production time simultaneously. They also used a multi-objective non-dominated sorting genetic algorithm to solve this problem. (Ebrahimi Zade et al., 2016) Using a heuristic algorithm and a new mathematical formulation for Solving Maintenance machine Scheduling Problem with Periodic Main Maintenance Using Genetic Algorithm to Robust Multi-Objective Optimization of Maintenance Scheduling Considering Engineering Insurance. such an important case that an increasing number of papers used machine learning for this problem because of the interpretability of machine learning models (Vollert et al., 2021). Maintenance and repair decision support systems have been boosted by the internet of things, big data, and machine learning. These systems play a vital role in ensuring the maintenance and reliability of equipment in industries by converting large data sets to useful knowledge and consequences (Ayvaz & Alpay, 2021; Chen et al., 2020; Schmitt et al., 2020). Organizations can implement these systems to monitor the soundness of industrial processes, optimize maintenance and repair programs, and receive immediate alerts on operational risks. Implementation of support decision systems allows organizations to minimize service costs, maximize job time, and improve productivity (Schwendemann et al., 2021).

The analysis of results obtained from previous studies indicates that the present paper complements the former studies on

failures and breakdowns in scheduling parallel machines and the effect of the failures and rules obtained from data mining on operations sequence. The extant study aims to achieve the problem objective by consideration of just-in-time delivery, energy consumption reduction, and cost decline.

### 3. Theoretical Foundations

#### 3.1. Scheduling parallel machines

This type of scheduling is the general form of the single-machine problem and a specific form of flexible workshop flow widely used in the real world. Most models considered in the literature are classified as offline scheduling problems; it means that all problem data (such as processing time, release time, and delivery times) are predetermined and can be considered in the optimization process.

#### 3.2. Data mining

Data mining is a set of important and practical tools that can be used to reduce waste, and identify and predict disruptions in production systems. Various algorithms with different approaches depending on the type of problem are used in data mining. The methods used in data mining are strongly correlated with machine learning algorithms. Machine learning algorithms are considered in different categories, including regression, classification, and clustering. Data mining methods are used to solve some problems, such as prediction of failures and detection of system reliability. Some of the formulas used to estimate systems' reliability have high computational costs. Moreover, it is not simple to formulate the reliability of a system or detect disruptions. Classification and modeling approaches can be used to design an instant and computational model for estimating the failure probability value, predicting future failure time, anticipating failure rate and level, and so forth.

Classification, clustering, or rule-based methods can be used for appropriate decision-making. The present paper used the LightGBM classifier algorithm. LightGBM is a Gradient boosting algorithm, which has a simple and understandable inference and makes some rules because it uses a tree-based learning approach.

#### 3.3. LightGBM Algorithm

This algorithm approaches a gradient-boosting framework, which performs based on the decision tree algorithms. (Ke et al., 2017) showed that This algorithm has been distributed and designed to be used for the following functions and advantages:

- High-speed training and efficiency
- Low use of memory
- Better precision
- Supporting parallel GPU-based computations
- Ability to manage large-scale data

This approach is a rapid and efficient gradient boosting that is designed based on the decision tree algorithms. LightGBM algorithm is usually used for ranking classification and many other tasks. This algorithm grows with the best fit through a leaf while other boosting algorithms perform through tree depth or its level instead of leaf, meaning it grows tree vertically while other tree-based learning algorithms grow horizontally. This process can be explained well in Figure 2. Therefore, when LightGBM grows on a leaf, it can create more loss decline compared to superficial algorithms. Hence, the results of LightGBM algorithms are more accurate and precise. Other boosting algorithms seldom generate such performance. As mentioned before, this is a high-speed algorithm; hence, it is called light.

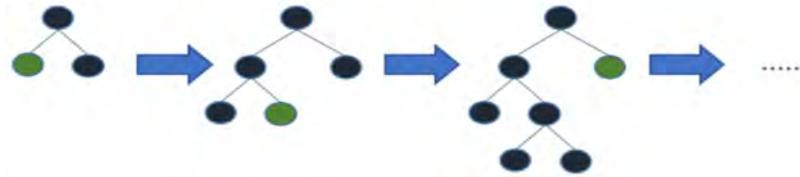


Figure 2. Formation process in LightGBM algorithm

### 3.4. Dataset

Data inserting and preprocessing is one of the significant data mining phases. The dataset was retrieved from <https://archive.ics.uci.edu/ml/datasets/> website, which comprises information related to numerical, sequential, and historical features that include 10000 observations. This study used 80% of the data as training data and

the remaining 20% as test data. In these observations, failures are represented as binary data (0, 1) where 1 means failure while 0 represents non-failure. Moreover, five characteristics also exist air temperature, process temperature, rotation speed, torque, and tool wear. Table 2 shows three lines of the problem dataset as an example.

Table 2. Problem's dataset

temperature	Process temperature	Rotational speed	torque	Tool wear	Machine failure
298/8	309/2	1425	53/9	135	0
298/8	309/2	1412	44/1	140	0
298/8	309/1	2861	4/6	143	1

Machine failures include four types:

- 1) Heat loss failure: heat loss causes process failure if the difference between air and process temperature is less than 8.6 kelvin and the instruments' rotation speed is lower than 1380 rpm. This case occurred for 115 data points.
- 2) Power outage: the power required for the process is obtained from torque multiplied by rotation speed (rad/s). If this power is less than 3500w or higher than 9000w, the process faces failure. This failure 95 times occurred in our dataset depicted in Figure 3.

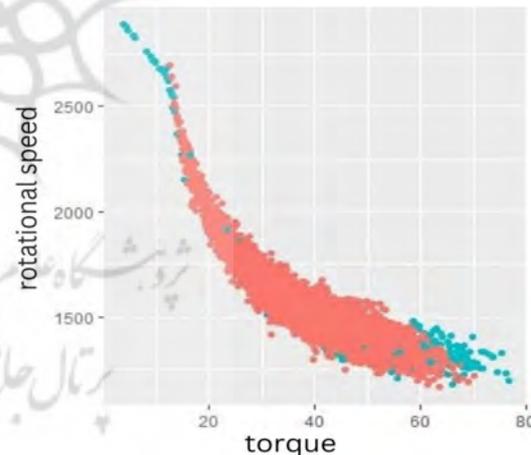


Figure 3. Graphical representation of the connection between three variables of rotation speed, torque, and machine failure

- 3) Overpressure failure: if the outcome of instrument wear and torque exceed 11000 newtons meter/minute, the process fails due to overpressure, which applies to 98 data points in this

dataset. Figure 4 depicts the connection between these three variables.

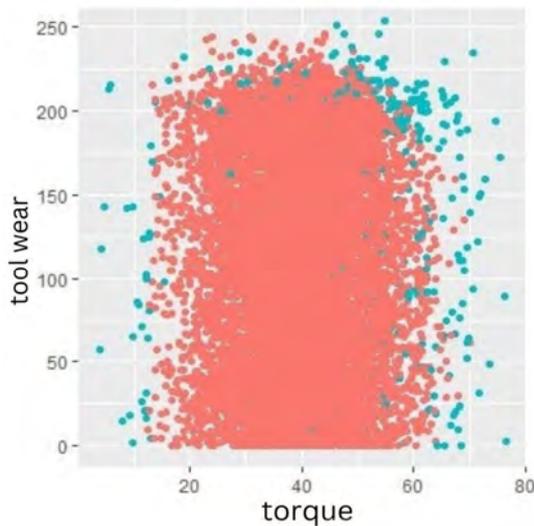


Figure 4. Graphical representation of the connection between three variables of tool wear, torque, and machine failure

- 4) Sudden failures: each process has a 0 and 1% chance if process parameters are not considered. This case occurred in 19 observations among data.

If at least one of the abovementioned failures occurs, the process faces failure, and the machine failure label is adjusted to 1, which applies to 339 data points. Therefore, it is not clear in the machine learning method which one of the failure modes has led to process breakdown. Therefore, the failure rate equals

3.39% the cost of the wrong classification of a false negative is 30 times greater than a false positive. However, all of the mentioned factors cause machine failure by adjusting the machine failure label on 1.

#### 4. Classification of failures

This analysis used 7 classifier algorithms to assess the accuracy of results and compare them: Logistic Regression, Decision Tree, Support Vector Machine (SVM), Naïve Bayes, Xgboost, Gradient Boost, and Light GBM. The mentioned algorithms were run through Python software, Sklearn Library, and a system with Ram 12 and Core i7 CPU.

##### 4.1. Evaluation Index

To analyze the results of the abovementioned algorithms, this study first obtained their confusion matrix and then measured four precision, accuracy, recall, and F1 metrics where TP represents the number of true positives, TN indicates the number of true negatives, FP shows the number of false positive, and FN is the number of false negatives. The confusion matrix is designed based on the machine learning models, and the results are divided into four categories. The confusion matrix consists of statistical results of real classification data and predictions conducted by these seven classifier algorithms. Table 3 depicts the confusion matrix.

Table 3.

Confusion Matrix

confusion matrix	predict	
	1	0
Real	1 True positives(TP)	True negatives(TN)
	0 False positives(FP)	False negatives(FN)

Precision metric indicates the percent of true “yeses” of the algorithm. This metric is measured as shown below:

$$\text{Precision} = \text{TP} / \text{TP} + \text{FP}$$

Accuracy is the most common, basic, and simple metric that measures the quality of

classification. Accuracy indicates the extent of classification's accurate detection in two categories. This parameter indeed indicates the number of patterns that have been detected accurately. The accuracy metric is defined as follows:

$$\text{Accuracy} = (\text{TP} + \text{TN}) / (\text{TP} + \text{FN} + \text{FP} + \text{TN})$$

Recall or rate of true positive responses is another metric. Recall means the ratio of positive options, which tests have detected accurately as positive samples. The recall is calculated based on the equation below:

$$\text{recall (TPR)} = \text{TP} / (\text{TP} + \text{FN})$$

F metric is widely used to evaluate the function of classifications. This metric is composed of two parameters of recall and precision. F metric is defined as follows:

$$\text{F-measure} = 2 * (\text{recall} * \text{Precision}) / (\text{recall} + \text{Precision})$$

Table 4 reports the function of all machine learning models in predicting failures. The results indicate that LightGBM outperforms other models. The accuracy and precision metrics of this algorithm equaled 98.55% and 85%, respectively; followed by the XGBoost model with accuracy and precision values of 98.3% and 78%, respectively.

Table 4

*Results of evaluated machine learning algorithms*

Algorithms	accuracy	Precision	recall	F1
logistic regression	71	97.35	27	39
Naïve Bayes	28	95.95	17	21
SVM	67	96.89	0.03	0.06
Gradient Boost	67	97.89	67	67
XGBoost	78	98.3	63	70
Light GBM	85*	98.55*	65	74
decision tree	73	98.25	70*	72*

#### 4.2. Hyper-parameters of the model

This part of the study introduces some meta-parameters that should be set for the LightGBM algorithm. There are numerous meta-parameters for LightGBM. In this case,

the number of trees and tree depth, learning rate, and booster type have been evaluated in this model by using cross-validation and k=10. Table 5 reports the optimal parameters used in the LightGBM algorithm.

Table 5

*Parameters of LightGBM model*

number of trees	number of depth	Learning rate	Boosting type
200	7	0.1	Multiple regression trees

In the graphical presentation of Figure 5, the tree associated with the algorithm has a depth of 4 so the rules obtained from it can be seen.

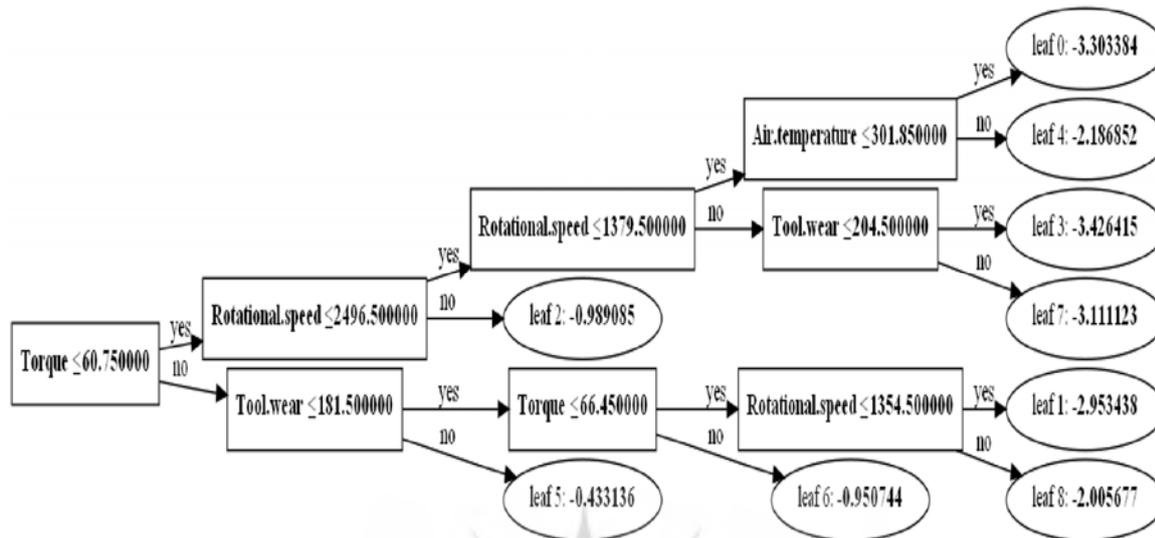


Figure 5. Graphical presentation of tree

For instance, the rules obtained from the decision tree algorithm indicate that the machine does not fail with a high probability if the rotation speed is less than 1380, tool wear is less than 181.5, and torque rate is less than 66.45.

## 5. Problem Definition

This part of the study formulated a mathematical model for the problem of parallel machine scheduling. In this model, repair time (the time between machine breakdown or failure and time to start it) is randomly measured using an exponential distribution. the reason is that this time distribution occurs when the failure of equipment is caused by the failures of one of its pieces. Moreover, the exponential distribution function does not have memory, so it can be used in any part of the problem. The time required to repair or serve equipment or pieces is called system maintainability. The time through which a device breaks down is a stochastic variable; hence the repair or maintenance time also is a stochastic variable.

If we assume that maintenance time follows exponential density function, maintenance rate is fixed, and its density function is obtained through equation (1).

$$f(x) = \lambda e^{-\lambda x} \quad x \geq 0 \quad (1)$$

where  $\lambda$  indicates the average number of maintenance and repairs in the time unit and  $\frac{1}{\lambda}$  indicates average maintenance time. Maximum Likelihood Estimation (MLE) was used to calculate  $\lambda$  and achieve an estimated value of  $\lambda$ . The exponential distribution is fitted on these data, lambda is  $\lambda$  is estimated assuming failures are independent of each other. Table 6 reports the number of failures per period. In this table, each period includes 30 days and each day consists of an 8-hour operation. Table 7 indicates the results obtained from the MLE method. We considered three machines in this example.

Table 6  
Number of failures per period

periods	Machine1	Machine2	Machine3
1	6	5	7
2	8	7	7
3	2	4	9
4	5	8	5
5	4	4	5
6	19	26	17
7	24	32	31

periods	Machine1	Machine2	Machine3
8	6	5	6
9	5	6	4
10	10	3	2
11	5	9	2
12	3	6	7
13	4	3	3
14	5	4	6

Table 7  
Statistical distribution related to the failure

Number of machines	Distribution parameter	Lower bound	Upper bound	The interval two failures (hours) machines
1	0.13	0.07	0.2	29
2	0.12	0.07	0.2	31
3	0.13	0.07	0.2	

According to the results of Table 7, the machine breaks down or fails almost after 30 hours. In a problem of machine maintenance and repair, the machine should serve regularly. In particular, we are interested to know how can begin machines' repair and maintenance cycles to meet all service requirements. Therefore, the scheduling can be accurately calculated based on the available average repair time in the mathematical model. On the other hand, we can estimate the machine unavailability times and compare them with data mining results. The following section includes the definition of sets and indexes, parameters, decision variables, and mathematical model relations and equations.

### 5.1. Problem's Assumptions

The problem is parallel machine scheduling with  $N$  jobs and  $M$  identical parallel machines. Various parameters exist for each piece:  $P_j$  indicates the natural processing time of job  $j$  that does not have any extra processing cost,  $d_j$  shows product delivery time,  $B_j$  indicates the penalty for delay in the time when the job  $j$  is completed lately and the  $\alpha_j$  shows penalty for early delivery when the job is completed before the due day. If processing time is

decreased or increased by one unit, the compressing ( $c_j$ ) or expansion ( $c'_j$ ) cost is increased or decreased. Each job can be compressed or expanded to its maximum reduction or enhancement extent. Moreover, initial rotation speed and torque are defined as parameters with constant values. It is aimed at determining the job sequence and the optimal value of job processing compression or expansion in each device simultaneously to minimize early and late delivery penalties, cost of compressing or expanding jobs, energy consumption, and machine failures. In this section, previous analyses of machine failure forecasts based on data mining are used to consider these disruptions in the model. First, data mining is used to find some rules about the rotation speed or torque causing failure. Moreover, unavailability or availability times of machines are inserted in the mathematical model, by using the forecast results. In this model, optimal rotation speed and torque are measured considering jobs' compression and expansion time separately. Hence, this model measures and optimizes energy consumption by considering the mentioned variables.

**5.2. Problem’s formula**

It is assumed that a set of N independent jobs exists  $N=\{j_1, j_2, \dots\}$  which is processed in identical parallel machines. Tables 8, 9, and 10 indicate the sets, indexes, parameters, and decision variables of the mathematical formula and model. Processing time can be compressed as much as  $x_j$  or expanded as much as  $x'_j$ , which requires compressing or expansion cost. Expansion or compression time of jobs is done by the machine. In this case, the job should be processed more rapidly in the compression case and vice versa. The increased or decreased velocity of the machine affects the failures and energy consumption of the machine.

Table 8

*Sets and indexes*

N		Number of jobs
M		Number of machines
K		Number of priorities
i, j	(i, j=1,2,...,N)	Jobs’ index
K	(k = 1,2,...,k)	Priorities’ index
m	(m=1,2,...,M)	Machines’ index

Table 9

*Parameters*

$p_j$	Normal job processing time
$c_j$	Cost of jobs compression
$c'_j$	Cost of jobs expansion
$\alpha_j$	Early delivery penalty
$\beta_j$	Late delivery penalty
$d_j$	Jobs delivery
$SPM_m$	The start point of machine unavailability
$FPM_m$	The endpoint of machine unavailability
$L_j$	Maximum extent of job compression
$L'_j$	Maximum extent of job expansion
$RS_j$	Rotation speed of each piece
$TO_j$	Torque of each piece
V	Cost per kWh
M	Large positive number

Table 10

*Decision variables*

$C_j$	Time of completing job j
$y_{jkm}$	1 if job j is assigned to machine m in priority k; 0 otherwise.
$E_j$	Early delivery time of each job
$T_j$	Late delivery time of each job
$X_j$	Compression time of each job
$X'_j$	The expansion time of each job
$U_{jm}$	one, if work of piece j starts before unavailability of machine m, 0 otherwise
$PM_j$	Power consumption of machine for each job
$S_j$	The start point of job j

**5.3. Mathematical model**

$$\text{Min } Z = \min (\sum_{j=1}^N (\alpha_j E_j + \beta_j T_j + C_j X_j + C'_j X'_j)) + (V * PM_j) \quad (1)$$

$$\sum_{m=1}^M \sum_{k=1}^K Y_{jkm} = 1 \quad \forall j \quad (2)$$

$$\sum_{j=1}^N Y_{jkm} \leq 1 \quad \forall k, m \quad (3)$$

$$Y_{j1m} (P_j + S_j - X_j + X'_j) \leq C_j \quad (4)$$

$$(\sum_{m=1}^M \sum_{k \geq 2} \sum_{i \neq j} Y_{ik-1m} Y_{jkm} C_i) + S_j + P_j - X_j + X'_j = C_j \quad \forall j \quad (5)$$

$$C_j - d_j = T_j - E_j \quad \forall j; \quad (6)$$

$$L_j \geq X_j \quad \forall j; \quad (8)$$

$$L'_j \geq X'_j \quad \forall j; \quad (10)$$

$$S_j + P_j \leq SPM_m \quad (11)$$

$$+ M(1 - u_{jm}) \quad j = 1, 2, \dots, n, m = 1, 2, \dots, m$$

$$S_j \geq FPM_m - M u_{jm} \quad j = 1, 2, \dots, n \quad m = 1, 2, \dots, m \quad (12)$$

$$RS'_j = \frac{RS_j \times P_j}{P_j - X_j + X'_j} \tag{13}$$

$$TO'_j = \frac{TO_j \times P_j}{P_j + X_j - X'_j} \tag{14}$$

$$LRS'_j \leq RS'_j \leq URS'_j \tag{15}$$

$$LTO'_j \leq TO'_j \leq UTO'_j \tag{16}$$

$$PM_j = \frac{2 * \pi * RS'_j * TO'_j}{60 + X_j - X'_j} * (P_j) \tag{17}$$

$$y_{jkm} \in \{0,1\} \quad \forall j, k, m ; \tag{18}$$

$$T_j, E_j, X_j, X'_j \geq 0 \quad \forall j ; \tag{19}$$

ensures that only one job can be processed with priority k in machine m and the first job in each machine is determined using constraint (4). Moreover, constraint (5) is used to identify the remaining job sequence in all machines. Constraint (6) defined the early and late delivery of job j. Constraints (8) and (10) specify the compression and expansion extent of each job. Constraints (11) and (12) indicate machine availability and unavailability times. Constraints (13) and (14) measure rotation speed and new torque. Constraints (15) and (16) apply the rules obtained from data mining, which create upper and lower limits for rotation speed and torque. Constraint (17) calculates electricity energy consumption. Constraints (18) and (19) provide 0 and 1 and non-negative logical requirements for decision variables.

### 5.4. Model Description

This section addresses the mathematical model, which integrally reduces energy consumption and failures simultaneously. This method forecasts failures and then schedules the machine and job. The following example used 5 jobs and two machines to clarify the considered approach. Table 11 reports values related to parameters. We used Figure 6 for a graphical illustration of scheduling.

Table 11  
Parameters' values

index	$p_i$	$D_i$	$L_i$	$L'_i$	$\alpha_i$	$\beta_i$	$c_i$	$c'_i$	$RS_i$	$TO_i$	$SPM_k$	$FPM_k$
1	4	5	1	1	2	2	2	2	1150	45	6	8
2	6	4	1	1	2	2	2	2	1220	52	0	0
3	5	8	1	1	2	2	2	2	1100	55		
4	7	15	1	1	2	2	2	2	1000	3		
5	5	13	1	1	2	2	2	2	1250	20		

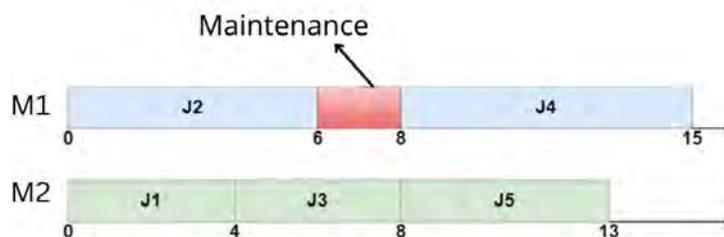


Figure 6. Scheduling five jobs and two machines

As it is seen, jobs start from 0-point, unavailability of machines 1 starts from 6, and machine becomes available from time 8 then. The objective function equals 32110, while the objective function caused by penalties equals 12 without considering the energy cost. In this case, job 2 has two units delivery delay so this job cannot be compressed because of the limitation in speed rise. The reason is that if there is one unit compression in the job then its rotation speed reached greater than 1400. Job

1 has one unit of early delivery, and one unit of job 3 is compressed in time 8.

In another example that is opposite of the previous example, we do not apply constraints on rotation speed and torque, so that optimal rotation speed and optimal torque can take any quantity. Figure 7 indicates scheduling this problem without any limitations on rotation speed and torque. Table 12 reports the values of the above mentioned parameters.

Table 12

Parameters' values

index	$p_i$	$D_i$	$L_i$	$L'_i$	$\alpha_i$	$\beta_i$	$c_i$	$c'_i$	$RS_i$	$TO_i$	$SPM_k$	$FPM_k$
1	5	5	1	1	2	2	2	2	1300	45	13	14
2	5	10	1	1	2	2	2	2	1220	52	0	0
3	5	19	1	1	2	2	2	2	1370	55	0	0
4	7	7	1	1	2	2	2	2	1300	38		
5	6	13	1	1	2	2	2	2	1250	32		
6	8	15	1	1	2	2	2	2	1380	50		
7	3	13	1	1	2	2	2	2	1245	30		
8	7	7	1	1	2	2	2	2	1260	41		
9	4	17	1	1	2	2	2	2	1320	35		

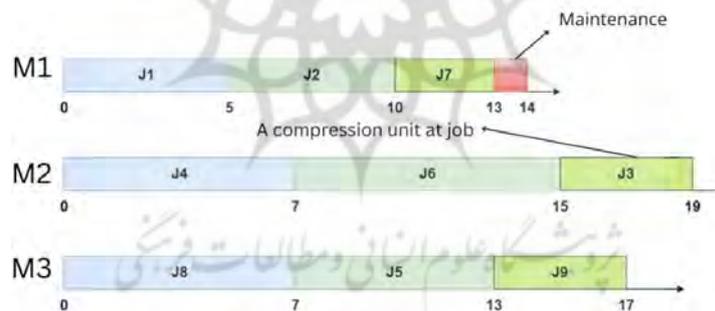


Figure 7. Scheduling without considering rotation speed and torque constraints

According to the results of scheduling, the machine can accelerate jobs by increasing rotation speed. However, the excessive increase may lead to machine and instrument failure. As can be seen in this example, job 3 has been accelerated by one unit and its processing time has reduced from five units to 4 units and torque has decreased from 55 to 45. Moreover, its rotation speed has increased from 1370 to 1713, which may cause machine failure, so these failures can cause financial

damage to the whole system. In general, it is concluded that data mining rules can prevent an excessive rise in rotation speed in the model and failures caused by such excessive decreases or increases. These rules also can optimize energy consumption. Moreover, these constraints reduce the solution time compared to the case in which, these constraints do not exist. Moreover, the completion time of machine 1 equals 13 after which, the machine is unavailable and repaired

and maintained. There is no delay or early delivery in this scheduling.

Moreover, we can compare the machine learning approach in failure detection and the empirical distribution of failures considering how these two approaches affect parallel machine scheduling. According to Table 7, every machine breaks down and needs repair and maintenance on average after 30 hours but machine learning algorithms predict failure by assessing the received data. Each machine fails with fails time interval, but the failure rate predicted by the LightGBM algorithm consists of the actual value. However, the fitted value of exponential distribution differs from the actual value. The latter indeed labels the times in which the machine is operating as a failure, and cannot detect the failure time causing a cost increase. Therefore, the machine learning model outperforms the empirical distribution model and imposes a lower cost.

## 5. Results

This study measured coefficients of early and late delivery fine, cost of compressing, and expanding jobs randomly with even distribution. The parallel machine scheduling parameter indicates the time when the machine becomes unavailable measured by the forecast. FPM is estimated after repairing or replacing instruments by using exponential distribution (the time when the machine becomes available). Processing time and job delivery time have also been considered randomly with even distribution. The results of the mathematical model have been compared for small dimensions (6-10 jobs) when job compression and expansion constraints are applied and are not applied. Table 13 reports the results of the comparison. In this case, modes A and B indicate whether the model is allowed to apply constraints related to upper and lower limits of rotation speed and torque, respectively. This model was run through GAMS with 12 Gb Ram and Core I7 CPU.

Table 13

*Results of mathematical model for this dimension*

Machine	jobs	case	Objective	Total earliness	Total tardiness	The number of possible failure points due to rotation speed and torque
2	6	A	36669	0	8	0
	6	B	36675	0	7	1
	7	A	39666	0	5	0
	7	B	39672	0	3	2
	8	A	50357	8	5	0
	8	B	50361	5	3	2
	9	A	50394	0	0	0
	9	B	50400	0	0	0
	10	A	59631	8	13	0
	10	B	59627	8	3	2
3	6	A	36660	0	3	0
	6	B	36665	0	1	2
	7	A	39664	0	3	0
	7	B	39671	0	2	1
	8	A	50339	0	3	0
	8	B	50345	0	0	1
	9	A	50394	0	0	0
	9	B	50394	0	0	0
	10	A	59589	0	0	0
	10	B	59599	0	0	1

Machine	jobs	case	Objective	Total earliness	Total tardiness	The number of possible failure points due to rotation speed and torque
4	6	A	36657	0	1	0
	6	B	36662	0	0	2
	7	A	39664	0	3	0
	7	B	39674	0	2	1
	8	A	50336	0	0	0
	8	B	50344	2	0	1
	9	A	50394	0	0	0
	9	B	50397	0	0	0
	10	A	59589	0	0	0
	10	B	59595	0	0	1
5	6	A	36657	0	1	0
	6	B	36664	0	0	1
	7	A	39664	0	3	0
	7	B	39667	0	3	0
	8	A	50336	0	0	0
	8	B	50342	0	0	1
	9	A	50394	0	0	0
	9	B	50396	0	0	0
	10	A	59589	0	0	0
	10	B	59595	0	0	1

A paired comparison test was done for two modes A and B through Minitab Software to confirm the model's results. This test assumes that approach A optimizes the process and minimizes the costs. The null hypothesis ( $H_0$ ) assumes that a difference exists between the mean values of two members of a paired society, i.e., the difference between two societies does not equal 0. The opposite hypothesis ( $H_1$ ) assumes there is not any difference between mean values. Mean values of objective function equal 47332 and 47337 for two modes A and B, respectively. The mean value of mode A is less than B, and the confidence interval of 95% of differences varies between -3.590 and -6.610 which does not include 0. Therefore,  $H_0$  is rejected; it means that the objective function's value of A is lower than mode B. Hence, the difference between A and B equals a negative value. The t-value measured for mean values' difference equaled -7.07, which is a minor quantity, and significance probability equals 0, which is less than the alpha value of 0.05. Therefore, the

value of an objective function that does not consider rotation speed and torque constraints is higher than the case in which, these constraints are not considered. Moreover, non-consideration of these constraints leads to higher machine failure.

## 6. Conclusion

This study aims at achieving a set of rules to predict machines' failure by using a machine learning algorithm. According to obtained results, a machine does not break down if it operates with a rotational speed < 1380 and torque < 60. Moreover, a comparison between algorithms indicated that the Light GBM algorithm with 85% precision outperformed other algorithms in predicting failures. The relevant results were reported in Table 4. Next, this study developed a mathematical model, applied these failure times, and achieved the optimal sequence of operations. This study was conducted to examine an integrated problem consisting of operations sequence, optimization of machine performance, and

energy consumption continuously. Finally, it was concluded that the developed mathematical model with 5 machines and 10 jobs for scheduling can be simply solved to achieve an optimal sequence. This sequence tends to reduce costs, energy consumption, and failures. Ultimately, it is suggested that further studies use this approach for other types of scheduling, including parallel machine scheduling and workshop flow. Moreover, meta-heuristic algorithms can be used for problems with larger dimensions.

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